The mass of (1) Ceres from its gravitational perturbations on the orbits of 9 asteroids

B. Viateau and M. Rapaport

Observatoire de Bordeaux, URA 352, CNRS, BP 89, F-33270 Floirac, France

Abstract. The mass of (1) Ceres is about half of the total mass of the main asteroid belt, and its long-term perturbations on the orbits of many solar system objects are important. For this reason, a very good knowledge of this mass is necessary. Although many determinations of the mass of Ceres have been made until now, the uncertainty remaining on its value is yet too high. A determination of the mass of Ceres, based on its gravitational perturbations on the orbits of 9 asteroids, is presented. All the available observations of the perturbed asteroids were used. In particular, for the asteroids observed by Hipparcos, the very accurate Hipparcos data were added to the ground-based observations. Other accurate observations, recently made with the CCD meridian circles of Bordeaux and Valinhos (near São Paulo, Brazil) observatories, were also included. The value obtained for the mass of Ceres, $(4.759 \pm 0.023)\ 10^{-10}$ M$_\odot$, is in good agreement with most recent results obtained by the other authors, and is a more precise value of this mass. In particular, this result shows that the value of the mass of Ceres recommended by IAU should be decreased by nearly 5 %.

Key words: minor planets, asteroids – astrometry – ephemerides – planets and satellites: individual: 1 Ceres

1. Introduction

Large asteroids induce non negligible and, sometimes, strong gravitational perturbations on the orbits of a great number of solar system objects: of main belt asteroids, but also of some planets. For example, the DE403 ephemerides from Jet Propulsion Laboratory (USA) take into account the gravitational perturbations of 300 minor planets, with estimates of their masses (Standish et al. 1995).

The masses of large asteroids are currently rather poorly known. The first attempt to determine a mass of an asteroid was made in 1966 by Hertz, who determined the mass of (4) Vesta from its gravitational perturbations on the orbit of (197) Arete (Hertz 1966). Thirty years later, about ten masses only have been determined, and, in most cases, with low accuracy.

2. Method

Before 1992, all the direct determinations of the masses of asteroids used their gravitational perturbations on the orbit of only one asteroid, i.e. the six orbital elements of the perturbed asteroid and the mass of the perturber were simultaneously corrected. In this case, the correlations between the mass of the perturbing asteroid and the orbital elements of the perturbed body, combined with the uncertainty of the observations used, especially the old ones, can induce a bias on the value obtained for the mass of the perturbing asteroid.

If several perturbed objects are simultaneously used to calculate the mass of the perturber, the correlations between the parameters are smaller, and the individual bias are averaged. To our knowledge, this method was used for the first time in Sitarski & Todorovic-Juchniewicz (1992). In this case, a large system is solved, in which corrections are simultaneously calculated for the mass of the perturbing body and for the osculating elements of all the perturbed asteroids.

Another method, easier to apply, is to make one individual determination of the mass for each perturbed asteroid, and to calculate the weighted mean of the individual values found for the mass. If we assume that there is no correlation between the individual determinations, it can easily be shown that the result obtained for the mass is strictly identical to the result obtained when solving a large system in which the osculating elements of all the perturbed asteroids and the mass of the perturber are simultaneously corrected. This last method was applied to the work presented in this paper.

On the other hand, the accuracy of the value found for the mass is linked with the quality of the orbit determined for the perturbed objects. For this reason, high quality recent observations are very useful to obtain a more accurate value of the mass because they enable us to improve the accuracy of the orbital data of the perturbed asteroids. This is the case of the observations of asteroids made by the satellite Hipparcos. 48 of the largest main belt asteroids were observed by Hipparcos between 1989 and 1993 with a very high accuracy (about 0.015"). Using these observations can therefore strongly improve the orbital data of the perturbed asteroids (see Sect. 3.1). This is also the case for the observations made since 1995 with the two CCD

Table 1. Asteroids used for the determination of the mass of Ceres. 'H' indicates the asteroids observed by Hipparcos. The close encounters with Ceres are detailed and classified with respect to the resulting maximal perturbation in R.A.$\times cos\delta$ (last column); d is the distance between both asteroids and v is their relative speed

Asteroid		Date of closest approach	d_{min} (AU)	v (AU/day)	$(d\,v^2)_{min}$	Pert. (")
(348) May		Sept. 1984	0.042	$4.7\ 10^{-4}$	$9.3\ 10^{-9}$	107
(203) Pompeja		Aug. 1948	0.016	$2.4\ 10^{-3}$	$9.2\ 10^{-8}$	78
(91) Aegina		Sept. 1973	0.033	$1.9\ 10^{-3}$	$1.2\ 10^{-7}$	62
(534) Nassovia		Dec. 1975	0.022	$1.6\ 10^{-3}$	$5.7\ 10^{-8}$	51
(2) Pallas	H	Oct. 1820	0.181	$7.2\ 10^{-3}$	$9.5\ 10^{-6}$	26
(32) Pomona		Nov. 1975	0.025	$2.7\ 10^{-3}$	$1.9\ 10^{-7}$	25
(324) Bamberga	H	Mar. 1944	0.020	$5.3\ 10^{-3}$	$5.8\ 10^{-7}$	12
(16) Psyche	H	Nov. 1975	0.198	$2.3\ 10^{-3}$	$1.1\ 10^{-6}$	10
(4) Vesta	H	Mar. 1893	0.186	$1.3\ 10^{-3}$	$4.3\ 10^{-8}$	5.5

meridian circles of Bordeaux (France) and Valinhos (near São Paulo, Brazil) observatories.

3. Determination of the mass of Ceres

3.1. The mass of Ceres

Ceres is the largest asteroid. Its mass is about half of the total mass of the main asteroid belt. For this reason, it is important that we have a precise value for its mass. This is not currently the case, although many attempts have been made since 1970 to determine this mass (see Table 7). For our part, we are involved in the determination of the mass of Ceres for a few years (e.g. Viateau & Rapaport 1995, Viateau 1995).

In the present paper, a determination of the mass of Ceres from its gravitational perturbations on the orbits of 9 asteroids, whose names are given in Table 1, is discussed. Some of these asteroids, like (91) Aegina, (203) Pompeja or (348) May are now frequently used for the determination of the mass of Ceres (e.g. Bowell et al. 1994, Carpino & Knezevic 1996). Among the other asteroids used, 4 were observed by Hipparcos. Using Hipparcos asteroids can be interesting for the determination of the mass of Ceres. Indeed, although the perturbation of Ceres on the orbits of these objects is not as large as for some other asteroids more commonly used (see Table 1), the Hipparcos asteroids are bright and, thus, can easily be observed. For this reason, their observations are in general abundant and of relative good quality, and span in most cases a very large time interval (Table 5). Moreover, the observations made by Hipparcos enable us to determine with a very high accuracy some of the orbital parameters of the perturbed asteroids. Thus, they contribute effectively, through the correlations between all parameters, to decrease the uncertainty on the semi-major axis of the orbit of these objects, closely linked with the uncertainty on the mass of Ceres. For each selected asteroid, an individual determination of the mass of Ceres was made (details will be given below). Attempts to determine the mass of Ceres by using other Hipparcos asteroids (e.g. (9) Metis) were also made but these asteroids were discarded because the uncertainty on the result obtained was too high.

3.2. Asteroids involved

Table 1 gives for each asteroid some details about its close encounter with Ceres. The effect of the gravitational perturbation induced by Ceres on the orbit of each asteroid was calculated for the dates of the observations of this asteroid with an assumed mass of Ceres equal to $5.0\ 10^{-10}\ M_\odot$. The value given in the last column is the maximal value of the effect of the perturbation found in right ascension $\times \cos\delta$. During an asteroid-asteroid encounter, the strength of the perturbation induced by the perturbing body on the orbit of the perturbed asteroid is inversely proportional to the quantity $d\,v^2$, where d is the distance between both asteroids (or impact parameter) and v is their relative speed.

3.3. Observations used

The observations of the perturbed minor planets were provided by the Minor Planet Center (USA) through its Extended Computer Service. For each asteroid, all the available observations were used. In the case of (203) Pompeja, old observations published in Goffin (1991), and not included in the MPC tape, were added.

For the asteroids observed by Hipparcos, the Hipparcos data were added. The Hipparcos observations had been separately reduced by the NDAC and FAST consortia, and for each observation, both positions had been provided to the scientific community (ESA 1997). It is reminded that the Hipparcos observations give only access to one coordinate, which is the abscissa on a Reference Great Circle (RGC) of the projected position of the asteroid. Since, for each observation, the positions given by FAST and NDAC are not independant, they cannot be used as separated observations as it is done for ground-based observations. Thus, for each observation, we obtained one single position by calculating the average of the positions given by the NDAC and FAST consortia.

Moreover, among the 9 asteroids used in the present paper, 6 were observed by the CCD meridian circles of Bordeaux and Valinhos observatories. These observations were made since 1995, as part of a collaboration between Observatoire de Bordeaux, France, and Instituto Astronomico e Geofisico of São Paulo, Brazil (Réquième et al. 1997). These observations were

Table 2. Details of the Bordeaux and Valinhos CCD meridian observations used. "B" and "V" indicate the number of observations from Bordeaux and Valinhos, respectively, M_V is the mean magnitude of the object in the V band, "R.A." and "Decl." indicate the standard deviation in arcsec of the whole set of meridian observations, in R.A. and Decl., respectively

Asteroid	dates	B	V	M_V	R.A.	Decl.
(32) Pomona	1996-97	14		11-12	0.053	0.043
(91) Aegina	1997	14	7	12-14	0.065	0.114
(203) Pompeja	1995-97	26	20	12-14	0.085	0.086
(324) Bamberga	1996-97	14		10-12	0.062	0.076
(348) May	1997	14	2	14	0.060	0.111
(534) Nassovia	1997	6		14	0.081	0.109

reduced with the Tycho catalog, and using Starnet proper motions for V mag > 8. For each asteroid, the number of CCD meridian observations from Bordeaux and Valinhos used and the standard deviation of their residuals after correction of the orbital parameters of the asteroid are given in Table 2. It can be seen that the precision of these observations is in general a little bit better in right ascension than in declination. The very good result obtained in the case of (32) Pomona in declination is explained by the fact that Pomona was moving slowly during the period of observation. Thus, most of the observed fields overlapped and the reduction procedure could be made with better accuracy.

3.4. Photocentre offset

The phase effect, i.e. the offset between the centre of light and the centre of mass of a solar system body is non negligible for the observations of the largest asteroids made by Hipparcos, due to the high accuracy of these observations. This effect has been recently studied by Hestroffer (1998). For the asteroids involved in our work, it has been found to be non negligible for (2) Pallas, (4) Vesta and, to a lesser extent, (324) Bamberga. However, Hestroffer (1998) pointed out the fact that, in the case of (324) Bamberga, the modelisation of the phase effect was unsatisfactory and its validity may be questioned. We therefore took this effect into account for (2) Pallas and (4) Vesta only.

The phase effect is given for the Hipparcos observations by:

$$\Delta v = cos\,(\theta_s - \theta)\,C(i)\,sin(i/2)\rho/2$$

where ρ is the apparent diameter, i is the solar phase angle and θ_s is the position angle in the tangent plane of the sub-solar point and θ is the position angle of the reference great circle corresponding to the observation.

The function $C(i)$ depends on the actual brightness distribution over the visible surface of the object. Following the conclusions of the study made by Hestroffer (1998), we adopted for $C(i)$ the values:

$$C(i) = 0.670 + 0.045\,i \; ; \; (i \leq 0.2)$$

$$C(i) = 0.686 + 0.037\,i \; ; \; (0.2 < i)$$

Table 3. Global rotation between the Hipparcos (ICRS) and FK5 Catalogues in equatorial coordinates. The orientation components are given at the epoch $T_o = $ J1991.25 (TT)

Orientation [mas]	Spin [mas/year]
$\varepsilon_{o_x} = -18.8 \pm 2.3$	$\omega_x = -0.10 \pm 0.10$
$\varepsilon_{o_y} = -12.3 \pm 2.3$	$\omega_y = +0.43 \pm 0.10$
$\varepsilon_{o_z} = +16.8 \pm 2.3$	$\omega_z = +0.88 \pm 0.10$

3.5. FK5-ICRS transformation

The Hipparcos positions of asteroids are given in the ICRS reference frame, as well as the positions of the Tycho stars used for the reduction of the CCD meridian observations. For homogeneity reasons, the ground-based observations were expressed in the ICRS reference frame. For this purpose, a transformation between the FK5 and the ICRS reference frames was applied on the coordinates $\mathbf{u}_{FK5}$ of all the ground-based observations but the CCD meridian observations made at Bordeaux and Valinhos, by means of a time-dependent rigid rotation $\boldsymbol{R}$:

$$\mathbf{u}_{ICRS} = \boldsymbol{R}\big(\boldsymbol{\varepsilon}(t)\big)\,\mathbf{u}_{FK5}$$

where $\boldsymbol{\varepsilon}(t) = \boldsymbol{\varepsilon}_o + \boldsymbol{\omega}\,(t - T_o)$, $T_o = $ J1991.25 (TT), and the components of the orientation and spin vectors (Mignard et al. 1997), are given in Table 3.

3.6. Dynamical model used

For each perturbed minor planet, the theoretical positions were calculated for each date of observation with the Bulirsh and Stoer numerical integration method (Bulirsh & Stoer 1966). The accuracy of the numerical integration procedure was approximatively 0.001 arcsec (Viateau 1995). The osculating elements given in the "Ephemerides of Minor Planets for 1995" (Batrakov 1994) were taken as initial conditions for all the asteroids (initial date JD = 2450000.5 TT). The integration of the motion of the perturbed minor planets took into account, as a standard procedure, the nine major planets but Pluto (VSOP82 theory, Bretagnon 1982) and, in addition, the perturbations of 7 large asteroids (plus Ceres). These asteroids are listed in Table 4 with the assumed value for their mass. The values of the masses were taken from the literature when possible, or, if not, were deduced from the mean diameter of the asteroids and an estimation of their mean density, as already made and detailed in Viateau & Rapaport (1997a). For Ceres, the initial value of $5.0\ 10^{-10}$ M$_\odot$ assumed was the value recommended by IAU Commission 20 at the IAU General Assembly of Buenos Aires in 1991 (West 1991).

For all the asteroids used for the determination of the mass of Ceres (the asteroids whose names are listed in Table 1), the gravitational perturbations on their orbit due to the 7 large asteroids are always small compared to the perturbation of Ceres. If it was not the case, the uncertainty on the masses of the 7 perturbing asteroids could have induced a non negligible bias on the value found for the mass of Ceres.

Table 4. Perturbing asteroids used

Asteroid	Mass $(10^{-10} \, M_\odot)$
(1) Ceres	5.0
(2) Pallas	1.2
(4) Vesta	1.35
(10) Hygiea	0.47
(11) Parthenope	0.026
(52) Europa	0.14
(511) Davida	0.18
(704) Interamnia	0.35

3.7. Data selection

An iterative procedure was used for each perturbed asteroid. At the first iteration, the residuals of the observations were calculated using the initial conditions given in Sect. 3.6. Since there were observations of very different epochs and, thus, of different accuracies, among the ground-based data, these data were separated into several groups with respect to the epoch of the observations. Each group was made up of observations showing residuals of about the same visual dispersion.

At every iteration, the standard deviation σ of the residuals was calculated for each group, and the observations giving residuals over $2.5 \, \sigma$ were eliminated. Weights corresponding to σ were given to the observations and a new solution was computed and used as initial conditions for the next iteration. In the case of photographic observations, since right ascension and declination are not independent of each other in the reduction procedure, both coordinates were rejected if one of them gave a residual over $2.5 \, \sigma$.

Iterations were made until convergence. Table 5 gives the total number of equations used (2 equations per ground-based observation, when not eliminated, and one per Hipparcos observation). It can be seen that in the case of (324) Bamberga, about 60 % of the Hipparcos observations, which showed quite scattered residuals, were eliminated. These observations, when not eliminated, prevented a good fit of the other Hipparcos data of (324) Bamberga.

3.8. Results and discussion

For each perturbed asteroid, the corrections for the six osculating elements and for the mass of Ceres were made by a classical least-squares method. Table 6 gives, for each asteroid, the condition number, the highest correlation coefficient between the mass and the six other parameters (i.e. the initial position and speed of the asteroid), and the value obtained for the mass of Ceres with its standard deviation. The explanation of the last column will be given below.

It first can be seen that, for all asteroids, the condition number is good. Moreover, there are not very high correlations between the mass of Ceres and the orbital parameters of the perturbed asteroids.

For 8 of the 9 asteroids used, a quite good agreement between the values of the mass is obtained, these values ranging between 4.54 and 4.96 10^{-10} $M_\odot$. This agreement gives confidence in the final result obtained for the mass (see below). The standard deviations on the individual values show that the global result will mostly depend on the values of the mass found for the 4 or 5 first asteroids listed in the Table 6. Looking more carefully at these particular asteroids, and especially at (348) May and (203) Pompeja shows that the differences between the values obtained for the mass, although not very high, are several times higher than the standard deviations on these values. However, it was noted during the iterations and also in the earlier determinations that the values of the mass of Ceres obtained with these two asteroids were very stable. On the other hand, it can be noticed that for the asteroids (4) Vesta, (324) Bamberga, (534) Nassovia and (32) Pomona, the internal coherence of the 4 values of the mass and their agreement with the final result are satisfactory with respect to the standard deviations obtained on these values.

The resulting global value of the mass of Ceres is $(4.759 \pm 0.023) \, 10^{-10} \, M_\odot$, and was obtained by calculating the weighted mean of the individual values of the mass. The standard deviation on this result is smaller than all other determinations of the mass of Ceres made until now (see Table 7). This standard deviation appears to be quite consistent, since all the preliminary results of this work obtained these last months with a smaller number of perturbed asteroids and less recent observations gave a mass of Ceres between 4.75 and 4.81 10^{-10} $M_\odot$ (e.g. Viateau & Rapaport 1997b, Viateau & Rapaport 1997c).

Our final value of the mass of Ceres is consistent with most of the recent results obtained by other authors. This value is in particular good agreement with the results found by Carpino & Knezevic (1996) and, to a lesser extent, by Bowell et al. (1994), which are among the more accurate results on the mass of Ceres. Our value appears to be like an "average" of these two results. The agreement is also not too bad with the value found by Standish et al. (1995) from the DE403 solution.

In our work, the orbits of the 9 asteroids used for the determination of the mass of Ceres were calculated by taking into account the gravitational perturbations of 7 large asteroids (see Sect. 3.6). In order to estimate the influence of these perturbations on the final result, we have also calculated the mass of Ceres without taking into account the perturbations of the 7 asteroids mentioned above. It can be seen, looking at the last column of Table 6, that the effects of these perturbations on the values found for the mass of Ceres, although not very strong in general, are often non negligible, especially in the case of (203) Pompeja, one of the better candidates for the determination of the mass. For this reason, the disagreement between the values of the mass found with Pompeja and (348) May could be explained by other non negligible asteroidal perturbations which are not yet taken into account. The global value of the mass of Ceres obtained when neglecting all the asteroidal

Table 5. Total number of equations used; "% E" means the percentage of eliminated equations

| Asteroid | Time interval | ground-based | | | Hipparcos | | |
	spanned	Initial	Final	% E	Initial	Final	% E
(2) Pallas	1802 - 1996	13164	9991	24	68	65	4
(4) Vesta	1827 - 1996	12682	9547	25	58	50	14
(16) Psyche	1852 - 1997	2814	1987	29	57	54	5
(32) Pomona	1864 - 1997	658	516	21			
(91) Aegina	1866 - 1997	742	568	23			
(203) Pompeja	1879 - 1997	544	430	21			
(324) Bamberga	1892 - 1997	1536	1229	20	77	30	61
(348) May	1892 - 1997	400	326	18			
(534) Nassovia	1904 - 1997	296	206	30			

Table 6. Results of the mass determination for each perturbed asteroid. The values of the mass of Ceres and the formal errors obtained are given in fourth and fifth column, classified with respect to the standard error, while the last column "No pert." gives the individual values of the mass obtained without taking into account any asteroidal perturbation

Perturbed asteroid	Condition number	Highest correlation coefficient	Mass	σ $(10^{-10} \, M_\odot)$	No pert.
(348) May	326	0.15	4.876	0.041	4.871
(203) Pompeja	67	0.14	4.626	0.046	4.759
(91) Aegina	547	0.43	4.961	0.061	4.988
(2) Pallas	431	0.31	4.546	0.064	4.658
(4) Vesta	322	0.48	4.692	0.088	4.641
(324) Bamberga	451	0.57	4.693	0.105	4.643
(534) Nassovia	115	0.34	4.673	0.137	4.603
(16) Psyche	200	0.63	5.203	0.228	4.510
(32) Pomona	337	0.56	4.819	0.238	5.175

perturbations is $4.797 \, 10^{-10} \, M_\odot$, thus the bias on the mass is nearly 2 times greater than the standard deviation on the result.

In the Tholen taxonomic classification, Ceres is a G-class asteroid (Tholen 1989), where the G class is a subclass of the C class (Tholen & Barucci 1989). Assuming the mean diameter of Ceres to be 932.6 ± 5.2 km (Millis et al. 1987, value obtained from a star occultation), our result gives a mean density for this asteroid of 2.23 ± 0.05 g/cm^3. This value is about 20 % higher than the 1.8 g/cm^3 mean density of C-class asteroids obtained by Standish et al. (1995) in the determination of the DE403/LE403 ephemerides.

Lastly, the most important result of this work is to show, in agreement with the majority of other authors, that the value of the mass of Ceres recommended by the International Astronomical Union, which is $5.0 \, 10^{-10} \, M_\odot$, appears to be too large by about 5 %. This drop has non negligible consequences on the calculation of the orbits of Mars and of many asteroids.

4. Conclusion

We have obtained a new value for the mass of Ceres from its gravitational perturbations on the orbits of 9 asteroids. This result was obtained by using a large number of old and recent weighted observations, especially the very high quality observations made by Hipparcos and the very accurate observations recently made with the CCD meridian circles of Bordeaux and Valinhos observatories. A particular care was taken to the asteroidal perturbations, the accuracy of the numerical integration method and the selection of the data, in order to try to avoid as many systematical effects in the final result as possible. The internal coherence of the individual values found for the mass of Ceres with each of the perturbed asteroids is quite good and the final result is in good agreement with the most accurate values obtained by other authors. Some differences noticed between the individual values obtained for the mass of Ceres may be caused by the poor quality of old observations of these asteroids, but may also be attenuated by adding future accurate observations of the corresponding perturbed asteroids, in particular in the case of (203) Pompeja, (348) May and (91) Aegina. Another way to explore is to try to take into account more asteroidal perturbations on the orbits of the studied asteroids, because some of these additional perturbations could modify the result found for the mass of Ceres in a non negligible way. Finally, this study shows that the value of the mass of Ceres currently recommended by IAU should be decreased by about 5 %.

Table 7. The current status of Ceres mass determination. σ indicates the formal standard deviation on the mass

Mass $(10^{-10}\,M_\odot)$	σ	perturbed bodies	Author
6.7	0.2	(2) Pallas	Schubart (1970)
5.1		(4) Vesta	Schubart (1971)
5.9	0.15	(2) Pallas	Schubart (1974)
4.99	0.09	”	Landgraf (1984)
5.0		”	Goffin (1985)
5.21	0.07	”	Landgraf (1988)
5.0	0.2	Mars	Standish & Hellings (1989)
4.9	0.15	(2) Pallas	Schubart (1991)
4.74	0.04	(203) Pompeja	Goffin (1991)
4.796	0.085	(203) & (348)	Sitarski & Todorovic-Juchniewicz (1992)
4.80	0.22	(348) May	Williams (1992)
4.85	0.06	6 asteroids	Bowell et al. (1994)
4.92	0.07	4 asteroids	Muinonen et al. (1994)
5.04	0.10	(2) Pallas	Viateau & Rapaport (1995)
4.67	0.09	5 asteroids	Carpino & Knezevic (1995)
4.64		DE403 solution	Standish et al. (1995)
4.26	0.09	5 asteroids	Kuzmanoski (1995)
4.88	0.45	(4) Vesta	Hilton et al. (1995)
4.78	0.06	(2) & (203)	Viateau (1995)
4.71	0.05	7 asteroids	Carpino & Knezevic (1996)
4.759	0.023	9 asteroids	this work

References

Batrakov Y.V., 1994, Ephemerides of Minor Planets for 1995, Institute of Theoretical Astronomy, St Petersburg

Bowell E., Muinonen K., Wasserman L.H., 1994, Asteroid mass determination from multiple asteroid-asteroid encounters. In: Abstracts for Small Bodies in the Solar System and their Interactions with the Planets (Mariehamn, Finland), 19

Bretagnon P., 1982, A&A, 114, 278

Bulirsh R., Stoer J., 1966, Num. Math. 8, 1

Carpino M., Knezevic Z., 1995, Asteroid mass determination: (1) Ceres. In: IAU Symp. 172, "Dynamics, Ephemerides and Astrometry of the Solar System", Paris, France

Carpino M., Knezevic Z., 1996, Determination of asteroid masses from mutual close approaches. In: Proc. of the First Italian Meeting of Planetary Science, Bormio, Italy

ESA, 1997, The Hipparcos and Tycho Catalogues. ESA SP-1200

Goffin E., 1985, see Landgraf (1992)

Goffin E., 1991, A&A, 249, 563

Hertz H.G., 1966, IAU Circular 1983

Hestroffer D., 1998, Limb-darkening of minor planets, A&A, submitted

Hilton J.L., Middour J., Seidelmann P.K., 1995, Prospects for Finding Asteroid Masses. In: IAU Symp. 172, Dynamics, ephemerides and astrometry in the solar system, Paris

Hilton J.L., 1996, The mass of (15) Eunomia. In: Asteroids, comets, meteors 1996, Versailles, France

Kuzmanoski M., 1995, A method for asteroid mass determination. In: IAU Symp. 172, "Dynamics, Ephemerides and Astrometry of the Solar System", Paris, France

Landgraf W., 1984, MPC 8665

Landgraf W., 1988, A&A, 191, 161

Landgraf W., 1992, In: Ferraz-Mello S. (ed.) Proc. IAU Symp. 152, Chaos, resonance and collective dynamical phenomena in the Solar System, Kluwer Academic Publishers, Dordrecht, p.179

Mignard F., Froeschlé M., Turon C., 1997, In: The Hipparcos and Tycho Catalogues, Vol. 3. ESA SP–1200, p. 415

Millis R.L., Wasserman L.H., Franz O.G., Nye R.A., Oliver R.C. et al., 1987, Icarus, 72, 507

Muinonen K., Bowell E., Wasserman L.H., 1994, In: Proc. Third International Workshop on Positional Astronomy and Celestial Mechanics (Lopez Garcia et al. eds, Valencia, Spain), 569

Réquième Y., Le Campion J.F., Viateau B., et al., 1997, Contribution of the CCD meridian circles to modern astrometry, In: Journées 1997 des Systèmes de référence spatio-temporels, Prague, Czech Republic

Schubart J., 1970, IAU Circular 2268

Schubart J., 1971, Asteroid masses and densities. In: Physical studies of Minor Planets (T. Gehrels ed.), NASA SP-267, 33

Schubart J., 1974, A&A, 30, 289

Schubart J., 1991, Astron. Rechen-Inst. Heidelberg, 41, 7

Sitarski G., Todorovic-Juchniewicz B., 1992, Acta Astronomica, 42, 139

Standish E.M., Hellings R.W., 1989, Icarus, 80, 326

Standish E.M., Newhall XX, Williams J.G., Folkner W.M., 1995, IOM 314.10-127, Jet Propulsion Laboratory, Pasadena, USA

Tholen D.J., 1989, In: Binzel R.P., Gehrels T., Matthews M.S. (eds.) Asteroids II. Univ. of Arizona Press, Tucson, p. 1139

Tholen D.J., Barucci M.A., 1989, In: Binzel R.P., Gehrels T., Matthews M.S. (eds.) Asteroids II. Univ. of Arizona Press, Tucson, p. 298

Viateau B., 1995, PHD Thesis, Observatoire de Paris, France

Viateau B., Rapaport M., 1995, AASS, 111, 305

Viateau B., Rapaport M., 1997a, A&A, 320, 652

Viateau B., Rapaport M., 1997b, In: Hipparcos Venice'97, ESA SP–402, p. 91

Viateau B., Rapaport M., 1997c, Contribution of Hipparcos data to the improvement of orbits and masses of asteroids. In: Journées 1997 des Systèmes de référence spatio-temporels, Prague, Czech Republic

West R.M., 1991, In: "Proceedings of the Twenty-first General Assembly, Buenos-Aires 1991", Transactions of the IAU 21B, Kluwer Academic Publishers, Dordrecht, p. 211

Williams G.V., 1992, In: Asteroids, Comets, Meteors 1991. Lunar and Planetary Institute, Houston, USA, p. 641