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The Historical Unravelling of the Diameters of the First Four Asteroids

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SUMMARY

At the dawn of asteroidal astronomy (*c.* 1801–1850) the diameter of Ceres was uncertain to within a factor of 10. Today the value is quoted to an accuracy of 0·6%. This paper discusses the way in which the estimates of the diameters of the first four numbered asteroids have improved in accuracy. Over the last two centuries disc micrometers have given way to photometers, and in their turn these have been replaced by filar micrometers, visual polarimeters, infrared photometers, speckle interferometers and occultation techniques.

1 INTRODUCTION

Science is supposedly 'exact' but this is the ideal case and unfortunately not the general rule. Measurements can be difficult to make and these difficulties lead to both random and systematic errors. The quest to measure the diameters of asteroids presents a perfect astronomical example of this problem. In the early nineteenth century, for example, two values were commonly quoted for the diameter of Ceres, these being 260 and 2613 km. Today the best estimate is 932·6 ± 5·2 km. In two centuries the uncertainty has changed from a factor of 10 to a mere 0·6%.

This paper concentrates on the first four numbered asteroids. 1 Ceres was discovered on the first day of the last century (1801 January 1) by Guiseppi Piazzi (1746–1826), an abbot of the Order of Theatines. At the time he was working at his observatory in Palermo (Sicily) and was engaged in the production of a better catalogue of the faint stars in the constellation of Taurus. Piazzi at first thought that he had found a new comet but after following the 9th magnitude object until February 11 he became convinced that it was a planet.

Johannes Kepler (1571–1630) had started the astronomical world worrying about the disproportionately large void in the planetary system between the orbits of Mars and Jupiter. His book *Mysterium Cosmographicum* (1596) contains the telling sentence "Inter Jovem et Martem interposui planetam". In 1772 Johann Daniel Titius (1729–1796) published the law that was to become known as the Titius–Bode Law. This again emphasized the existence of a gap between Mars and Jupiter. Baron Francis Xaver von Zach (1754–1832), the court astronomer at the Ernestine Observatory at Seeburg near Gotha, had even gone so far as to organize (in 1800) a society of 24 astronomers (the Society for the Detection of a Missing World—jocularly termed the celestial police) to explore each half-hour of the zodiac and search

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for the unknown planetary 'gap filler'. On hearing of Piazzi's 'comet' Johann Elert Bode (1749–1826), the director of the Berlin observatory, was convinced that the missing planet had been found.

Only Piazzi saw Ceres in 1801. When the news of its discovery reached northern Europe bad weather had set in and Ceres had moved behind the Sun; Piazzi had fallen ill and even after superior conjunction no-one could find it. Karl Friedrich Gauss (1777–1855) introduced a 'least squares' analysis of the positional data in order to calculate its orbit and thus produced an epheremis. It was only after the publication of this ephemeris that Ceres was rediscovered, this happening on the first birthday of its discovery when it was seen independently both by von Zach and by Heinrich Wilhelm Matthaus Olbers (1758–1840), the latter being a doctor in Bremen. Gauss had calculated that Ceres had a semi-major axis of 2·767 AU. The solar system gap was filled and other planets were not expected in that region. Olbers, however, discovered 2 Pallas on 1802 March 28. Near Bremen, in Lilienthal, there was a large private observatory belonging to Johann Hieronymus Schröter (1745–1816), who was the chief magistrate of the area and a well-known amateur astronomer. (Unfortunately the observatory was sacked and burned to the ground by the Napoleonic army in 1813 April.) Olbers often visited. After the orbit of Pallas had been calculated much was made of the fact that the periods of Ceres and Pallas (1681·4 and 1686·3 days respectively) were so similar. Olbers believed that these bodies must be the left-over pieces of some larger planet that had broken up, and consequently he expected many more pieces to be found in the future. Asteroids were thus very much in the news and Karl Ludwig Harding (1765–1834), an astronomer at the Lilienthal Observatory, found 3 Juno on 1804 September 1. After a systematic search Olbers struck again with 4 Vesta on 1807 March 29.

Even though Olbers continued searching energetically until 1817 nothing else was found. Nearly 40 years passed until Karl Hencke, the postmaster of Driesen in Neumark, found 5 Astraea on 1845 December 8. He had started his asteroid search in 1830.

A hundred asteroids had been found by 1868, the discovery rate then accelerating, numbers reaching 500, 1000, 2000, 3000, 4000, and 5000 by 1903, 1923, 1960, 1981, 1989 and 1992 respectively.

2 ASTEROID DIAMETERS 1801–1845

Celestial mechanics dominated early asteroidal studies and astronomers took great pains to delineate their orbital parameters, if for no other reason than to ensure that the newly-found minor planets were not subsequently lost. The first 'astrophysical' measurement was of their size. This presented astronomers with a very difficult problem because even Ceres at opposition has a diameter that subtends only 0·7 arcsec. Sir William Herschel (1738–1822) tackled the problem immediately after learning of their discovery and concluded that Ceres and Pallas had diameters of 259 and 238 km respectively (see Herschel 1802). He made much of the fact that Ceres "is of a remarkably small size, deviating much from that of all the primary planets". Herschel used his 7-foot focal length, 6·3-inch aperture reflector and a lucid disc micrometer. The latter consisted of a small illuminated disc that could be moved towards and away from the telescope. The telescope eyepiece was such that the asteroid could be seen with one eye while the disc could be viewed with the other. Herschel found that Ceres had an angular diameter of 0·40 arcsec. Using the 10-foot reflector he obtained a diameter of 0·38 arcsec. Pallas was found to have an angular diameter of 0·13 arcsec. Using the rather crude orbital parameters of the time Herschel concluded that Ceres had a real diameter of 161·6 miles and Pallas had a diameter of either 147 or 110·3 miles, the latter result being obtained on a night when the "greatest possible distinction was obtained". The appearance of the two asteroids varied considerably from night to night. They seemed to be surrounded by a hazy "coma" the diameter of which varied with the seeing. A typical report from 1802 read,

> May 1 ... 20-feet reflector; power 477. I see Pallas well, and perceive a very small disk, with a coma of some extent about it, the whole diameter of which may amount to 6 to 7 times that of the disk alone.

Herschel insisted that the asteroidal coma was different from that of a typical comet. This is not surprising because the asteroid's coma was clearly produced by deficiencies in Herschel's optical system. Using both the same instrument and magnification three days later he wrote

> I viewed Ceres, in order to compare its appearance with regard to haziness, aberration, atmosphere, or coma, whatever we may call it, to the same phenomena of the fixed stars; and found that the coma of the asteroid did not much exceed that of the stars.

Measurements made of the diameters of the asteroids by the German astronomer Johann Hieronymus Schröter (1745–1816) gave values that were a factor of 10 larger than Herschel's. Schröter (1811) quoted diameters of 2613, 3380 and 2290 km for Ceres, Pallas and Juno. These values nearly brought the asteroids into the same size-league as Mercury (4850 km) and Mars (6790 km). The great difference between the two asteroidal size estimations arose, according to Schröter, from "Dr H. observing with his projection-micrometer at too great a distance from his eye, and measuring only the middle clear part of the nucleus" (see Carey, 1831).

The astronomy books of the period prevaricated, some opting for the small diameter values, others the large, and many simply quoting both. Guy (1819) has the diameter of Ceres as 2834 km, Pallas 3669 km and Juno "nearly equal to that of Ceres". Aspin (1825) quotes diameters for Ceres as 262 km (Herschel) and 2614 km (Schröter), Pallas, 129 and 3378 km, Juno "is variously stated at 2293 and 2486 km" and Vesta "only 383 km, or according to some no more than 262 km". Tomlinson (1840) writes "Ceres is about 2832 km in diameter", "Pallas is stated to be rather smaller than the Moon in size", Juno diameter 2414 km, and "Vesta ... is supposed to be not more than 383 km in diameter, though this is not yet considered as settled". Dick (1846), dispensing with Herschel's appellation "asteroids" refers to them as "four new planets" and thought Ceres to have a diameter of 2614 km "but its atmosphere is reckoned at about 1086 km in height". The diameter of Pallas is given as 3378 km "and [is] consequently nearly the size of our Moon", Juno is "estimated by certain German astronomers at 1425 English miles (2293 km)" and Vesta at 444 km diameter—"but it is probable, from a variety of circumstances, that it is considerably larger".

First Steps to Astronomy and Geography (Hatchard and Son, Piccadilly,

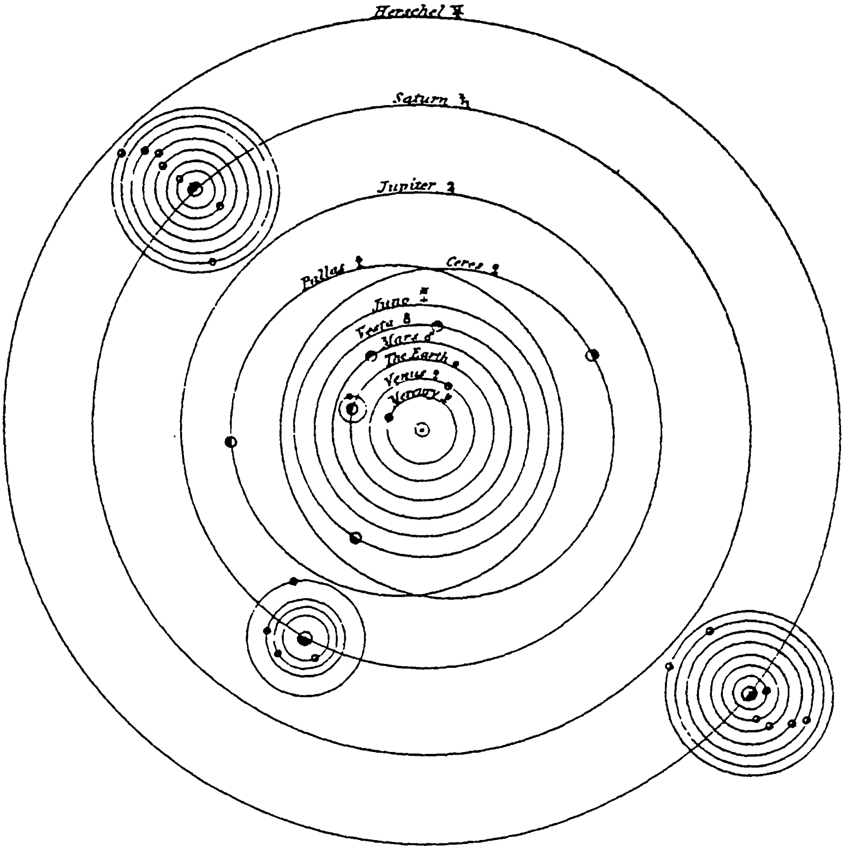

FIG. 1. The Orbits of the Planets, this being Plate 4 in *First Steps to Astronomy and Geography* (1828, J.Hatchards, Piccadilly, London). (The author is unknown, but the book is written for a juvenile audience and consists of a series of questions addressed by a mother to her two children William and Elizabeth.) At the time only four asteroids were known and, possibly due to the large values for their diameters obtained by Schröter, they figured prominently in contemporary illustrations of the solar system. One rather unusual aspect of this figure is that the planet Herschel (i.e. Uranus) is depicted as having seven satellites. Most books of the period (i.e. Guy 1819, Aspin 1825, Dick 1846 etc) listed six. By the mid 1850s the number had increased to eight. Following Arago (1857, op. cit. p. 506) and Chambers (1867) these can be listed as follows.

	Name	Discoverer	Date of discovery	Sidereal period (days)
1	Ariel	W.Lassell	1847 Sept. 14	2·52
2	Umbriel	O.W.Struve	1847 Oct. 8	4·14
3		F.W.Herschel	1790 Jan. 18	5·89
4	Titania	F.W.Herschel	1787 Jan. 11	8·71
5		F.W.Herschel	1794 March 26	10·96
6	Oberon	F.W.Herschel	1787 Jan. 11	13·46
7		F.W.Herschel	1790 Feb. 9	38·08
8		F.W.Herschel	1794 Feb. 28	107·69

William Lassell, in a letter of 1853 January 11, was "fully persuaded that either Uranus has no other satellites than numbers 1, 2, 4 and 6 of the list above, or if it has they remain to be discovered." Lassell was right. Only the named satellites in the list turned out to be "true" satellites of Uranus. The fifth "true" satellite, Miranda, was discovered by G.P.Kuiper in 1948 (at a magnitude of 16·5, on a 4 min photographic exposure with the 82-inch McDonald Observatory Telescope, see Kuiper, 1949) and to reach the seven in the figure we had to await the flyby of the spacecraft Voyager 2 in January 1986.

TABLE I

The recorded values of the diameters (in km) of Ceres, Pallas, Juno and Vesta as a function of time

Observer/recorder		Date	1 Ceres	2 Pallas	3 Juno	4 Vesta
Herschel		1802	259	235	—	—
Schröter		1811	2613	3380	2290	—
Lamont		—	—	1073	—	—
Mädler		—	—	—	584	467
Stampfer		1856	365	277	180	367
Stone		1867	315	275	200	344
Pickering		1879	—	269	151±7	513±17
Harrington		1883	—	—	—	840
Flammarion		1894	350	270	200	400
Barnard		1895	781±87	490±118	195	390±46
Hamy		1899	—	—	—	390
Barnard		1901	706±84	—	—	347±70
Widorn	P	1967	850	500	—	390
Dollfus*		1967/9	—	921±256	—	435±73
Dollfus		1970	770±40	490±50	200±50	420±35
Dollfus		1971	770	(700)	195	410
Allen	R	1971	1160±80	—	290±20	570±10
Matson	R	1971	1000±100	530^{+100}_{-175}	—	600±60
Cruikshank	R	1973	1080±80	550±50	250±24	540±40
Morrison	R	1973	1020±100	—	—	580±60
Veverka	P	1973	1220^{+120}_{-240}	660±110	—	580^{+70}_{-90}
Bowell	P	1973	1060±130	600±75	—	550±70
Zellner	P	1974	1050	570	222	490
Zellner	P	1976	914	573	233	496
Hansen	R	1976	1173±104	754±34	318	602±51
Hansen	R	1977	1019	597	275	538
Morrison	R	1977a	1018±43	585±57	241	531±15
Zellner	P	1977	957	—	—	558
Morrison	C	1977b	1003	608	247	538
Worden	SI	1979	—	673±55	—	550±23
Schubart	C	1979	987±150	538±50	—	544±80
Bowell	C	1979	1025	583	249	555
Wasserman	O	1979	—	538±12	—	—
Millis	O	1981	—	—	267±5	—
Dunham	O	1983	—	523±5	—	—
Lebofsky	R	1986	936±12	532±7	—	—
Millis	O	1987	932·6±5·2	—	—	—
Tedesco	C	1989	913±43	523±20	244±12	501±24

SI = speckle interferometry, O = occultation, R = radiometric, P = polarimetric, C = combination.

* See Dollfus (1971).

London, 1828), a book for a juvenile audience written as a dialogue between a mother and her two children William and Elizabeth, posed the question "how many planets, then, are there altogether?". Favouring the large asteroid option, Elizabeth replies, "Eleven: Mercury, Venus, the Earth, Mars, Vesta, Juno, Ceres, Pallas, Jupiter, Saturn, and Herschel". (The debate as to whether Uranus should be called Georgium Sidus or Herschel, who discovered the planet on 1781 March 13, was still raging.) Figure 1 emphasizes the supposed prominence of four asteroids known at that time.

Schröter, "an energetic, but over-imaginative astronomer" (see Waterfield 1938), also noted that Ceres was "surrounded by an extensive and dense atmosphere" (Carey 1831). This clearly throws light on to the telescopic problems of the day and to quote Agnes Clerke (1885)

> there is no good reason to suppose that any of the minor planets possess atmospheres. The aureoles seen by Schröter to surround Ceres and Pallas have been dissipated by optical improvements.

Further measurements of asteroidal diameters are recorded by François Arago (1857) and the values obtained by J.H.Mädler (using the Dorpat refractor) and J.Lamont are given in Table I. Arago noted that the angular diameter of Vesta, obtained by Schröter, was 0·488 arcsec. Systematic and random errors are not quoted but these must have been very large. Arago reports that Herschel tried to measure the angular diameter of Vesta during its 1807 May opposition and found this diameter to be less than 10 per cent that of Uranus. Herschel also found that the opposition angular diameters of Ceres and Pallas were 0·35 and 0·24 arcsec respectively and Arago questioned how measurements of such small angles could be made with any accuracy or consistency, considering the typical angular size of the stellar images seen with the instruments of the day.

3 ASTEROIDAL DIAMETERS 1846–1893

The next step in the 'diameter saga' relied on photometry. The assessment of stellar and planetary brightness had been placed on a firm footing by three events, these being the introduction of the logarithmic magnitude scale by N.Pogson in 1854, the recording of stellar magnitudes in the famous star catalogue *Bonner Durchmusterung* (produced by F.W.A.Argelander and published in 1863) and the introduction of the first visual stellar photometer by J.C.F.Zöllner in 1861.

The visual brightness, b, of an asteroid depends on, among other things, its mean area of cross-section (which is proportional to the square of the equivalent diameter, D, this being the diameter of a sphere with the same surface area), the visual albedo, p_V, (the fraction of the received light that it reflects), the inverse square of its distance from both the Sun, r, and the Earth, Δ, and its phase. (Just like the Moon, asteroids can be seen when they are 'full', 'gibbous', 'quartered' or 'crescent shaped'.) So

$$b \propto p_V D^2/(\Delta r)^2. \tag{1}$$

For two objects, A and B, of the same albedo the brightness ratio is given by

$$\frac{b_A}{b_B} \propto \left(\frac{D_A \Delta_B r_B}{D_B \Delta_A r_A}\right)^2 = 10^{0\cdot4(m_B - m_A)}, \tag{2}$$

where m_A and m_B are their mean opposition magnitudes. For the average opposition sighting one can put $r = a$ and $\Delta = (a - 1)$, where a is the semi-major axis. Thus Eqn (2) gives the ratio between the diameters of the two objects. Typical values of the parameters in Eqn 2 are given in Table II. Needless to say it can be seen from Table II that the assumption that the first four asteroids all have the same albedo is way off the mark.

TABLE II

Relevant asteroidal parameters

	1 Ceres	2 Pallas	3 Juno	4 Vesta
Mean opposition magnitude, B(a, o)*	7·56	8·64	9·67	6·85
Absolute magnitude B(1, o)*	4·11	5·18	6·43	4·31
Rotation period (h)†	9·075	7·811	7·210	5·342
Amplitude (mag.)†	0·04	0·03–0·16	0·14–0·22	0·12
Semi-major axis (AU)	2·767	2·771	2·670	2·362
Eccentricity	0·097	0·180	0·218	0·097

* Gehrels (1970).
† Lagerkvist *et al.* (1989).

Today people often follow in the footsteps of Zellner & Bowell (1977) and compute diameters from equations of the form:

$$2\log(D/2) = 5{\cdot}642 - 0{\cdot}4\ V(1,0) - \log p_V, \qquad (3)$$

where $V(1,0)$ is the absolute magnitude, this being the magnitude the asteroid would have if it were observed when it was 1 AU from both the Sun and the Earth and at zero phase (i.e. when 'full'). As the way in which asteroidal brightness varies as a function of phase is known, real-time observations of asteroidal brightness can be used to evaluate $V(1,0)$. All that is then required to obtain the diameter [from Eqn (3)] is the albedo. Unfortunately, albedos were not known during the 1846–1901 time period under consideration, so guesses had to be made.

Stampfer of Vienna (1856) carried out photometric observations and his results are given in Table I. E.J.Stone (1867) took the mean opposition magnitudes of the 71 asteroids that he found listed in the 1868 Supplement to the *Nautical Almanac* and, by assuming that they all had the same albedo, he was able to calculate the relative diameters. To produce real diameters he compared his results with the diameters obtained for Ceres (315 km) by Sir W.Herschel and for Pallas (275 km) by J.Lamont. Sadler (1895) notes that "if these figures (i.e. the ones obtained by Herschel and Lamont) are incorrect, and I must say I have never been able to find them anywhere, the deduced diameters of the 69 others in his paper are also in error". Sadler also wonders whether Stone took Bruhn's value of 275 km for the diameter of Pallas in error for Lamont's actual value of 1073 km. Stone's results are given in Table I.

A similar photometric procedure was adopted by Pickering (1879). He, however, made his own measurements of the asteroidal apparent magnitudes. Because the asteroids are fairly close to Mars in the solar system, Pickering also assumed that they had the same albedo as that planet. The value of the assumed albedo was not given but Zöllner (1865), just over a decade before, had quoted a value of 0·2672, in comparison to values of 0·1736, 0·6238 and 0·4981 for Moon, Jupiter and Saturn respectively. Typical values today would be 0·154 for Mars and 0·112, 0·44 and 0·47 for Moon, Jupiter and Saturn (see Allen 1973).

Harrington (1883) found that Vesta had an albedo of about 0·1 ("much like that of the Moon and Mercury") and was about 520 miles (840 km) in diameter. He also stressed the fact that Vesta varied in brightness and wrote "we may presume that she has a very rough surface and rotates on her axis". Flammarion (1881) seemed to try to combine estimates of asteroidal angular diameter (when near opposition) with measurements of their brightness and his results are also given in Table I.

A hint as to the confusion that existed at the time can be gleaned from the short note in the *English Mechanic* published by Sadler in 1895. "I have thought it might be of interest to some of your readers if I collected the principal results obtained for the first four asteroids. The results marked with an asterisk are photometric and not deduced from micrometrical measurements." His results have been converted to kilometres *Ceres*—Schröter 2527, Herschel 261, Argelander 362*, Bruhns 365*, Galle 637, Knott 1014 and Barnard 837. *Pallas*—Schröter 3259, Herschel 196 (and 134 and 113 "almost incredibly small"), Lamont 1073, Argelander 254*, Bruhns 275*, Pickering 269*, Barnard 439. *Juno*—Schröter 2221, Argelander 169*, Bruhns 185*, Stone 200*, Pickering 151*. *Vesta*—Schröter 536, Mädler 467, Argelander 435*, Bruhns 370*, Stone 344*, Secchi 724, Tacchini 1416, Millosevitch 1014, Pickering 513*, Barnard 381.

This list of diameters was reproduced by Barnard (1895). He noted that "the uncertainty of the photometric method in determining the dimensions of these bodies is well shown by the large differences in the results derived by even this moderate range of assumed albedo".

4 ASTEROIDAL DIAMETERS 1894–1960

A major advance was made in the subject between 1894 and 1895 when E.E.Barnard (1895, 1900, 1901, 1902) made filar micrometer measurements of the diameters of Ceres, Pallas, Juno and Vesta using both the 36-inch Lick and 40-inch Yerkes refractors (see Table I). Barnard's values were subsequently accepted for well over half a century.

Barnard (1895) obtained the albedos of the asteroids by combining his angular diameters with other people's measurements of the mean opposition magnitudes of the asteroids. He noted in passing that when it came to the magnitudes "these various determinations by different observers differ in some cases by nearly an entire magnitude". Barnard recorded that the ratios between the albedos of Ceres, Pallas, Juno and Vesta and the albedo of Mars were 0·67, 0·88, 1·67 and 2·77 respectively. "This great difference of albedo is in no wise unreasonable. If one examines the four bright satellites of Jupiter he will find, in as closely a connected system as they constitute, similarly great differences of albedo." He also noted that the "very wide range of albedo among the asteroids themselves (was) at least as great as that found among the planets".

Barnard stressed that asteroids on average had an albedo that was 1·45 times that of Mars and that "this value will be a far safer standard to use in the determinations of the dimensions of the other asteroids by photometric methods than that previously adopted".

Up to the time of Barnard's micrometer measurements with the large refractor telescopes Vesta was considered to be the largest asteroid simply because it had the greatest brightness. The correct assessment of the relative

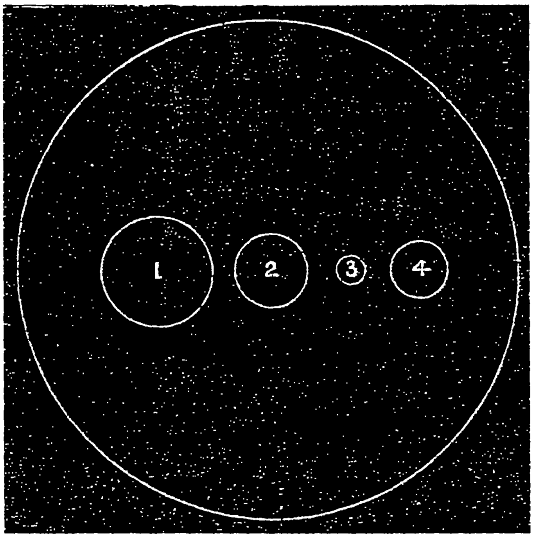

FIG. 2. A reproduction from Barnard (1895), showing the relative sizes of the first four numbered asteroids 1 Ceres, 2 Pallas, 3 Juno and 4 Vesta. The enclosing circle represents the size of the Moon.

albedos of the first four asteroids promoted Ceres to the top of the diameter list.

Barnard (1895) concluded by writing,

> I think it is scarcely necessary to add that all previous attempts at direct measurements of the diameters of these small bodies were made with instruments inadequate to deal satisfactorily with such minute quantities as the asteroidal diameters. They are far more difficult to deal with than the four bright moons of Jupiter.
>
> To make the relative sizes of these four asteroids apparent at once to the eye, I give a diagram made from my measures.

This diagram is reproduced as Figure 2. Moulton (1907) recorded how the combination of Barnard's diameter measurements with Muller's photometric magnitudes indicated that the actual albedos of Ceres, Pallas, Juno and Vesta were 0·18, 0·24, 0·45 and 0·75. Disagreeing with Barnard he goes on to say, "it is difficult to account for these great differences in bodies which we should expect to find much alike".

Barnard (1902) noted that the angular diameters of Ceres, Pallas, Juno and Vesta, reduced to a common distance from Earth of 1 AU, were 1·060, 0·675, 0·266 and 0·531 arcsec respectively. Dollfus (1971) writes that "these filar micrometer measurements are difficult to make when the disks are only slightly larger than the image of the diffraction pattern blurred by atmospheric seeing; the accuracy is necessarily poor, especially for Juno; the last decimals given are not significant".

An interferometer with a double slit in the wave-front was used by Hamy (1899), at the focus of the 60-cm coudé refractor of the Paris Observatory, but he only observed Vesta, obtaining a diameter of 400 km.

In the 1940s and 1950s a double-image micrometer was used by several observers, the results being reported by Dollfus (1971). Again he emphasizes the problems, stressing that the measurement of the size of an image that is only two to three times the effective resolving power of the telescope is subject to large uncertainties coupled with systematic errors that are difficult to evaluate. The fact that the diameter of Pallas quoted by Dollfus was twice the value measured by Barnard cast doubts on the accuracy of all the available determinations of asteroidal diameter.

A *diskmeter* was also used. This device (see Carmichel 1953) produced a small artificial bright disc-like image in the field of view of the telescope, this image having adjustable brightness, colour, blurring and diameter. This was first compared with the image of a stellar point source and then, changing all its parameters with the exception of the blurring, it was compared with the image of an asteroid. By this means the angular diameter of the asteroid could be measured and Kuiper did this at the prime focus of the 5-m Mount Palomar reflector (see Dollfus 1970).

Dollfus (1971) summarized all the asteroidal diameter determinations available in 1970 and his mean values are given in Table I.

Worden & Stein (1979) applied the speckle interferometry technique to the measurement of asteroid diameters (see Table I). On 1977 February 3, when Pallas and Vesta were measured, they had angular diameters of 0·73 ± 0·6 and 0·47 ± 0·2 arcsec respectively. This technique has been pushed even further and Drummond & Hege (1989) now consider Pallas and Vesta to be triaxial ellipsoids, Pallas having dimensions of 537 ± 29 km, 488 ± 11 km and 485 ± 11 km and Vesta having dimensions 566 ± 11, 531 ± 15 and 467 ± 15 km.

This was the first time that the largest three asteroids were regarded as being non-spherical. Following Cole (1986) the material inside an asteroid will show a restricted rigidity depending on the nature and strength of the applied stresses. For asteroids larger than a certain diameter the gravitational force will overcome the rigidity. Over a period of time this material will flow and the shape of the asteroid will adjust and become spherical. The diameter value that divides spherical and non spherical asteroids is usually found to be around 200 km.

5 MODERN ASTEROIDAL DIAMETERS

The most direct method of measuring asteroidal diameters is by the use of stellar occultations. Each asteroid casts a shadow in the light of every star in the sky. If this shadow intersects Earth then multiple observations of the occultation of the star in question can lead to a delineation of the cross-sectional shape of the asteroid perpendicular to the line of sight to the occulted star. If only one observer sees the occultation only one arc is obtained across the asteroidal cross-section. This case was illustrated by Taylor (1962). But as the number of observers increases the number of arcs increases and the cross-sectional shape becomes better defined.

The first four asteroids are massive and it is normally assumed that their gravitational field is sufficient to pull them into an approximately spherical shape. Thus, if the cross-section is known together with the asteroid–Earth distance, the diameter quickly follows. On 1978 May 29 the star SAO 85009 was occulted by Pallas, and Wasserman *et al.* (1979) analysed the data from seven observations. The cross-sectional shape was approximated to that of an ellipse with major and minor axes of 279·5 ± 2·9 and 262·7 ± 4·5 km

TABLE III

The albedos of the asteroids

		1 Ceres	2 Pallas	3 Juno	4 Vesta
Moulton	1907	0·18	0·24	0·45	0·75
Cruikshank	1973	0·06±0·1	0·08±0·02	0·14±0·02	0·21±0·03
Veverka	1973	0·05	0·06	—	0·19
Zellner	1974	0·068	0·084	0·197	0·278
Zellner	1976	0·068	0·082	0·181	0·271
Hansen	1977	0·050	0·079	0·127	0·235
Morrison	1977	0·054	0·074	0·151	0·229
Wasserman	1979	—	0·103	—	—
Tedesco	1989	0·10±0·01	0·14±0·01	0·22±0·02	0·38±0·03

respectively. The mean diameter of the asteroid was given as 538±12 km, this being an accuracy of 2 per cent.

On 1983 May 29 Pallas occulted the star 1 Vulpeculae. One hundred and forty arcs were obtained across the asteroid and the mean diameter was quoted as being 523±5 km (see Dunham *et al.* 1983). The 3 Juno occultation that took place on 1979 December 11 led to a mean diameter of 267±5 km. Millis *et al.* (1981) gave the semi-major and semi-minor axes of the observed cross-section as 145·2±0·8 and 122·8±1·9 km respectively. The stellar occultation by Ceres on 1984 November 13 led to a mean diameter of 932·6±5·2 km. Millis *et al.* (1987) concluded that Ceres was an oblate spheroid of equatorial diameter 959·2±4·8 km and polar diameter 906·8±9·0 km.

Millis & Dunham (1989) reviewed occultation measurements and their results are given in Table I. They concluded that the first four asteroids, Ceres, Pallas, Juno and Vesta, were approximately spherical and that the surface was rough on a scale of a few kilometres. Viscous interiors coupled with a fast spin rate (see Table II) would lead to the polar diameters being less than the equatorial diameters and so perfect sphericity is not to be expected.

The vast majority of asteroids have not been seen to occult stars. Here the diameter is still calculated by combining photometric measurements of the asteroidal magnitude with both radiometric and polarization approaches to the calculation of the albedo (see Morrison 1977b). Morrison's values for the albedos and diameters of the first four asteroids are given in Tables III and I respectively. Zellner & Bowell (1977) pointed out at the time that "the radiometric albedo scale gives somewhat lower albedos and larger dimensions than the polarimetric calibrations". The values listed in Table I do not support this idea. Other combinations of results from different techniques were produced later and these were published as the Tucson Revised Index of Asteroid Data (TRIAD), see Bowell, Gehrels & Zellner (1979). This data set was revised in 1989 (see Tedesco 1989).

The surface of a large asteroid is thought to consist of a microscopically intricate, dark, porous material in which multiple scattering is not dominant. Light reflected by this surface is polarized and the curve of polarization as a function of phase angle is of a similar form to that of the Moon. For low phase angles the polarization is negative, dropping to a minimum value for a phase angle just below 10°. The polarization then increases to zero at around a phase angle of 20°, and becomes positive beyond that (see for example Dollfus & Zellner 1979). What is important in the context of asteroidal diameter measurements is the fact that the slope of the linear part of the polarization curve beyond the inversion angle is found to be inversely correlated with the surface albedo. This relationship, the so-called Umov effect, can be calibrated in the laboratory by using pulverized meteoritic and lunar samples. Using this technique Veverka (1971a, b), for example, found that Vesta had an albedo of $0{\cdot}25 \pm 0{\cdot}07$ and thus a mean diameter of 512 ($+38$, -22) km. The polarization technique of albedo estimation has been applied to many of the bright asteroids. Table I contains asteroid 'polarization' diameters for the first four asteroids that have been obtained by Widorn (1967), Veverka (1973), Bowell *et al.* (1973), Zellner, Gehrels & Gradie (1974), Zellner & Gradie (1976) and Zellner *et al.* (1977).

Infrared photometry also leads to the estimation of asteroidal albedos and thus asteroid diameters. A fraction, p_V, of the incident visual solar radiation is reflected by the asteroid, where p_V is the visual albedo. The complementary fraction of the radiation, $(1-p_V)$, is absorbed and then reradiated in the infrared. As an asteroid has no atmosphere and (at the present time) a negligible internal heat source, its surface is in equilibrium with the incident sunlight. Thus the infrared magnitude is a function of $(1-p_V)$. Given that the visual magnitude is a function of p_V, the measurement of both magnitudes gives the albedo, and from Eqn (3), the diameter. A problem arises due to the rotation of the asteroid. This causes some of the thermal radiation to be emitted on the night side and this obviously reduces the signal received at Earth. At a wavelength of 10 μm the large asteroids provide signals of comparable intensity to those from the brightest stars and the flux at Earth could be measured (in 1970) to an accuracy of 10 per cent. The 'infrared' diameters reported by Allen (1971) are given in Table I. Schubart & Matson (1979) confirm the 10 per cent accuracy (see Table I). Table I contains other asteroid 'infrared' diameters obtained by Cruikshank & Morrison (1973), Matson (1971), Morrison (1973, 1977a, c) and Hansen (1976, 1977).

Unfortunately, things had hardly improved a decade later. The infrared measurements that lead to the measurements of asteroidal albedos still produce results that rely on the thermal model, and Lebofsky & Spencer (1989) still report that diameters obtained in this way are accurate to no better than 10 per cent. Notice that the *IRAS* albedos listed by Tedesco (1989) are about a factor of 1·7 greater than the ground-based estimates. This is due to the accurately-determined occultation diameters being used as a calibration.

6 CONCLUSIONS

The speckle interferometric work reported by Drummond & Hege (1989) indicated that the mean diameter of the projected disc of Vesta varies between extremes of 498 and 548 km. Assuming that Vesta was a triaxial ellipsoid of axes a, b, and c it was found that $(abc)^{1/3}$ was 520 km. In 1989 the radiometric diameter of Vesta was quoted in the TRIAD file as 530 km, the *IRAS* diameter (Matson *et al.* 1986) was given as 501 ± 24 km and the polarimetric diameter as 579 km.

For Pallas the projected disc was found to have a diameter that varies between 503 and 547 km, depending on whether one is viewing the asteroid from an equatorial or polar direction. Averaging speckle interferometer data with occultation data Drummond & Hege (1989) gave a value for $(abc)^{1/3}$ of 524 km. The radiometric diameter (Lebofsky *et al.* 1986) was 532 km and the *IRAS* diameter (Matson *et al.* 1986) was 523 ± 20 km.

So the diameters of the first four asteroids, Ceres, Pallas, Juno and Vesta, are now known to an accuracy of about ±5 per cent, this being a vast improvement on the precision of the measurements made during the first decade of the nineteenth century when their diameters were uncertain by a factor of over ten. In the future the term 'mean diameter' will become less important when applied to these four asteroids because occultation and speckle interferometric work will lead to a more precise definition of the slightly-ellipsoidal shapes.

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